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Inertia Effects in Hydrostatic Thrust Bearings

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This paper includes the predominant inertia terms in an analysis of hydrostatic thrust bearings. The influence of centripetal accelerations on the distribution of pressure is found to be considerable. For parallel-surface bearings of constant film thickness the inertia effects are found to be detrimental to load capacity. In a stepped bearing however, correct location of the step can result in an increased load capacity at speed. No increase in load capacity can result from inertia effects if the step radius is less than 0.4508 of the bearing radius. A consequence of the inclusion of inertia terms in the analysis is the existence of a velocity component in the axial direction. Even in the parallel-surface bearing considered a fluid element is found to move towards the rotating surface as it spirals through the clearance space.

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Inertia Effects in Hydrostatic Thrust Bearings

D. DOWSON

NOMENCLATURE

h = minimum oil film thickness

Re = Reynolds number $\left(\rho \frac{Uh}{\eta} \frac{h}{R}\right)$

r, θ, z = cylindrical co-ordinates

r_o = radius of lubricant supply hole

r_1 = radius of step

R = outside radius of bearing

r' = radius ratio (r/R)

P = total load capacity

Q = volume flow rate

S = inertia parameter $\left(\frac{3}{20} \rho \frac{R^2 \Omega^2}{p_o}\right)$

u, v, w = velocity components in r, θ, z directions

U, V, W = representative velocities

u', v', w' = velocity ratios $\left(\frac{u}{U}, \frac{v}{V}, \frac{w}{W}\right)$

p = pressure

p_o = representative pressure

p' = pressure ratio (p/p_o)

$\bar{p}$ = mean pressure across the oil film

α = film thickness ratio, $\frac{\text{film thickness in recess}}{\text{minimum film thickness}}$

ρ = density of lubricant

η = viscosity of lubricant

Ω = rotational speed

INTRODUCTION

One of the customary assumptions made in the theory of lubrication is that the influence of inertia terms in the equations of motion is negligible compared with the effect of the viscous terms. This assumption is clearly valid if the appropriate Reynolds number $Re \ll 1$. Brand (1)[1] carried out an order-of-magnitude analysis of two of the equations of motion involved in thrust bearings and he concluded that the Reynolds-number condition could be written in the form

$$\rho \frac{\Omega h^2}{\eta} = 1.$$

[1] Underlined numbers in parentheses designate References at the end of the paper.

An investigation of the relative importance of pressures produced by centrifugal inertial effects and normal hydrostatic action in tapered thrust pads was carried out by Kingsbury (2) on his electrolytic tank. For the bearing geometry considered Kingsbury found that the ratio of the two pressures was about 1/800 at a rotational speed of 1000 rpm.

In recent years the question of inertia effects has again been raised owing to the increase in bearing operating speeds. Most of the work has been concerned with journal bearings employing liquid and gaseous lubricants. By averaging the inertia terms across the oil film Osterle, Chou, and Saibel (3) were able to calculate the modified pressures in journal bearings. They found that even at the limiting condition for stable laminer flow inertia effects would only modify the pressures by about 4 per cent.

The investigations by Kingsbury and Osterle, Chou and Saibel were concerned with inertia produced modifications to hydrodynamic pressure distributions which are in turn speed dependent. For this reason the percentage change in pressure due to inertia effects may still be small at high speeds whilst the actual modification to the pressure may be considerable.

Shaw and Strang (4) have suggested that inertia-induced pressures may account for the observed improvement in the performance of parallel surface bearings at high speed.

This paper is concerned with inertia effects in hydrostatic thrust bearings. In this type of bearing the pressure distribution predicted in the absence of inertia effects is independent of the surface speeds of the bearing components. Consequently the relative importance of inertia effects can be much greater than in hydrodynamic bearings.

GENERAL ANALYSIS

If body forces are neglected the Navier-Stokes equation of motion for an incompressible isoviscous fluid in cylindrical co-ordinates take the form,

$$\rho\left[\frac{Du}{Dt} - \frac{v^2}{r}\right] = -\frac{\partial p}{\partial r} + \eta\left[\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right]$$

$$\rho\left[\frac{Dv}{Dt} + \frac{uv}{r}\right] = -\frac{1}{r}\cdot\frac{\partial p}{\partial \theta} + \eta\left[\nabla^2 v + \frac{2}{r^2}\frac{\partial u}{\partial \theta} - \frac{v}{r^2}\right]$$

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$$\rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \eta \nabla^2 w$$

For problems possessing axial symmetry the steady state representation of the above equation is,

$$\rho\left[u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right] = -\frac{\partial p}{\partial r} + \eta\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right]$$

$$\rho\left[u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r}\right] = \eta\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2}\right]$$

$$\rho\left[u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right] = -\frac{\partial p}{\partial z} + \eta\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right]$$

The continuity equation for an incompressible fluid in cylindrical co-ordinates is,

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial ur}{\partial r} + \frac{1}{r}\cdot\frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0$$

For axial symmetry and the steady state this reduces to,

$$\frac{1}{r}\frac{\partial ur}{\partial r} + \frac{\partial w}{\partial z} = 0$$

Now the equations can be written in dimensionless form by writing,

$$u = u'U \qquad v = v'V \qquad w = w'W$$

$$r = r'R \qquad z = z'h \qquad p = p'p_0$$

where the representative quantities U, R, p_o, and so on, are selected such that the primed variables are always $\leqslant 1$.

The equations now become

$$\rho\left[\frac{U^2}{R}u'\frac{\partial u'}{\partial r'} + \frac{WU}{h}w'\frac{\partial u'}{\partial z'} - \frac{V^2}{R}\cdot\frac{v'^2}{r'}\right]$$
$$= -\frac{p_0}{R}\frac{\partial p'}{\partial r'} + \frac{\eta U}{R^2}\left[\frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'}\frac{\partial u'}{\partial r'} + \left(\frac{R}{h}\right)^2\frac{\partial^2 u'}{\partial z'^2} - \frac{u'}{r'^2}\right]$$

$$\rho\left[\frac{UV}{R}u'\frac{\partial v'}{\partial r'} + \frac{WV}{h}w'\frac{\partial v'}{\partial z'} + \frac{UV}{R}\frac{u'v'}{r'}\right]$$
$$= \frac{\eta V}{R^2}\left[\frac{\partial^2 v'}{\partial r'^2} + \frac{1}{r'}\frac{\partial v'}{\partial r'} + \left(\frac{R}{h}\right)^2\frac{\partial^2 v'}{\partial z'^2} - \frac{v'}{r'^2}\right]$$

$$\rho\left[\frac{UW}{R}u'\frac{\partial w'}{\partial r'} + \frac{W^2}{h}w'\frac{\partial w'}{\partial z'}\right]$$
$$= -\frac{p_0}{h}\cdot\frac{\partial p'}{\partial z'} + \frac{\eta W}{R^2}\left[\frac{\partial^2 w'}{\partial r'^2} + \frac{1}{r'}\frac{\partial w'}{\partial r'} + \left(\frac{R}{h}\right)^2\frac{\partial^2 w'}{\partial z'^2}\right]$$

$$\frac{U}{R}\frac{1}{r'}\frac{\partial u'r'}{\partial r'} + \frac{W}{h}\frac{\partial w'}{\partial z'} = 0 \qquad (1)$$

The primed quantities are all of the same order of magnitude, and if only the predominant viscous terms are retained the equations of motion are

$$R_e\left[u'\frac{\partial u'}{\partial r'} + \frac{W}{U}\cdot\frac{R}{h}w'\frac{\partial u'}{\partial z'} - \left(\frac{V}{U}\right)^2\frac{v'^2}{r'}\right] = -\frac{p_0 h^2}{\eta U R}\cdot\frac{\partial p'}{\partial r'} + \frac{\partial^2 u'}{\partial z'^2}$$

$$R_e\left[u'\frac{\partial v'}{\partial r'} + \frac{W}{U}\cdot\frac{R}{h}\cdot w'\frac{\partial v'}{\partial z'} + \frac{u'v'}{r'}\right] = \frac{\partial^2 v'}{\partial z'^2} \qquad (2)$$

$$R_e\left[u'\frac{\partial w'}{\partial r'} + \frac{W}{U}\cdot\frac{R}{h}w'\cdot\frac{\partial w'}{\partial z'}\right] = -\frac{p_0 h}{\eta W}\cdot\frac{\partial p'}{\partial z'} + \frac{\partial^2 w'}{\partial z'^2}$$

where the Reynolds number

$$R_e = \rho\frac{Uh}{\eta}\left(\frac{h}{R}\right)$$

It is convenient to define the Reynolds number in this way since the appropriate selection of U enables the analysis to be applied to radial flow between a pair of stationary surfaces or to the combined radial and circumferential flow between a pair of rotating surfaces. With the equations of motion in the foregoing form the order of magnitude of the inertia terms can readily be estimated.

In the case of stationary surfaces and pure radial flow the inertia and viscous terms will be of equal importance when Re = 1. In this case the representative radial velocity U could be taken as $Q/2\pi r_o h$.

For rotating surfaces the last inertia term in the first equation of motion may well predominate, and in this case inertia and viscous terms will be equally important when Re $(V/U)^2 = 1$. It should be noted that if $R\Omega$ is taken as the representative velocity in the radial and circumferential directions this condition reduces to Brand's result of

$$\frac{\rho\Omega h^2}{\eta} = 1$$

In general the Reynolds number Re will be considerably less than unity in hydrostatic thrust bearings since U is usually small and $h \ll R$. Also since $W/U = O(h/R)$ from equation (1) all the inertia terms in the second and third equations of motion can be neglected. Similarly two inertia terms in the first equation of motion can be discounted and we finally obtain

$$\left.\begin{aligned} \frac{\rho v^2}{r} &= -\frac{\partial p}{\partial r} + \eta\frac{\partial^2 u}{\partial z^2} \\ 0 &= \eta\frac{\partial^2 v}{\partial z^2} \\ 0 &= -\frac{\partial p}{\partial z} + \eta\frac{\partial^2 w}{\partial z^2} \end{aligned}\right\} \qquad (3)$$

The second of equations (3) can be integrated directly, and with the boundary conditions z = 0, $v = 0$ and z = h, $v = r\Omega$ we find,

$$v = r\Omega\left(\frac{z}{h}\right) \qquad (4)$$

If the third equation of motion is integrated twice we obtain

$$\int p\,dz = \eta w + Az + B \qquad (5)$$

where A and B are functions of r only. When z = 0 the integral in equation 5 and w are both zero, and hence B = 0. Since w is also zero when z = h we have,

$$A = \frac{1}{h}\int_0^h p\,dz = \bar{p}$$

where $\bar{p}$ can be considered as the mean pressure across the oil film. Clearly $\bar{p}$ is a function of r only.

On differentiating equation (5) with respect to z and introducing the above expression for A we obtain

$$p = \bar{p} + \eta\frac{\partial w}{\partial z} \qquad (6)$$

From equation (6) it will be noted that the pressure at any point in the oil film will differ from the mean pressure by an amount which is dependent upon the local axial velocity gradient. For radial flow between stationery parallel surfaces $\partial w/\partial z$ is zero everywhere, but it will be shown that this is not true if inertia effects are considered in the case of rotating surfaces.

If the result expressed in equation 4 is introduced into the first of equations 3 and the expression for p in equation 6 is used to eliminate $\partial p/\partial r$, we find,

$$\frac{\partial \bar{p}}{\partial r} = \eta\left[\frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 w}{\partial r\,\partial z}\right] + \rho r\Omega^2\left(\frac{z}{h}\right)^2$$

The second viscous term in this equation can be written in terms of u and r from the continuity equation to give,

$$\frac{\partial \bar{p}}{\partial r} = \eta\left[\frac{\partial^2 u}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial ur}{\partial r}\right)\right] + \rho r\Omega^2\left(\frac{z}{h}\right)^2$$

In this form it can be seen that the first viscous term is of order $(R/h)^2$ times the second, and since $R \gg h$ the second term may be neglected. Thus

$$\frac{\partial \bar{p}}{\partial r} = \eta\frac{\partial^2 u}{\partial z^2} + \rho r\Omega^2\left(\frac{z}{h}\right)^2 \qquad (7)$$

The derivation of this equation does not preclude the existance of pressure gradients and hence velocities in the axial direction.

Since $\bar{p}$ is a function of r only equation (7) can be integrated directly with respect to z. When the boundary conditions z = 0, u = 0 and z = h, u = 0 are introduced the following expression for the radial velocity is obtained

$$u = -\frac{z(h-z)}{2\eta}\cdot\frac{\partial \bar{p}}{\partial r} + \frac{\rho r\Omega^2 z(h^3-z^3)}{12\eta h^2} \qquad (8)$$

Now since w = 0 when z = h we see from the continuity equation that,

$$\int_0^h \frac{1}{r}\cdot\frac{\partial ur}{\partial r}\cdot dz = 0$$

and hence by substituting for u from equation (8)

$$\frac{d}{dr}\left(r\frac{d\bar{p}}{dr}\right) = \frac{3}{5}\rho r\Omega^2 \qquad (9)$$

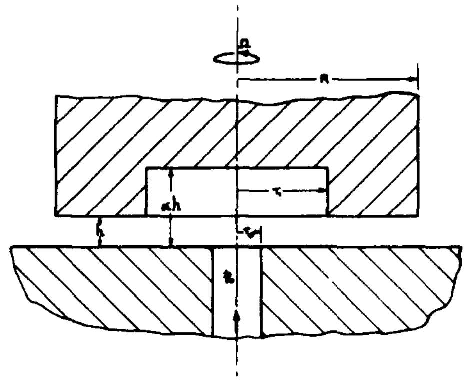

Fig. 1 Hydrostatic step bearing

The general form of the pressure distribution which is obtained from equation (9) before boundary conditions are applied is,

$$\bar{p} = \frac{3}{20}\rho r^2\Omega^2 + C\log_e r + D \qquad (10)$$

where C and D are integration constants.

The volume rate of flow of lubricant through the bearing can be obtained by evaluating the integral,

$$Q = 2\pi\int_0^h r u\,dz$$

It is found that Q is related to C by the expression

$$Q = -\frac{\pi h^3 C}{6\eta} \qquad (11)$$

SOLUTION FOR THE STEPPED PARALLEL SURFACE HYDROSTATIC THRUST BEARING

Pressure Distribution

For a bearing of the form shown in Fig.1 we need expressions for the pressure distribution on either side of the step.

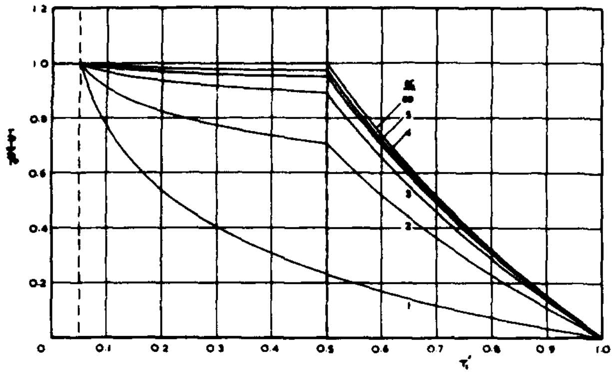

Fig. 2 Pressure distribution in a stationary hydrostatic step bearing. $r_0' = 0.05$

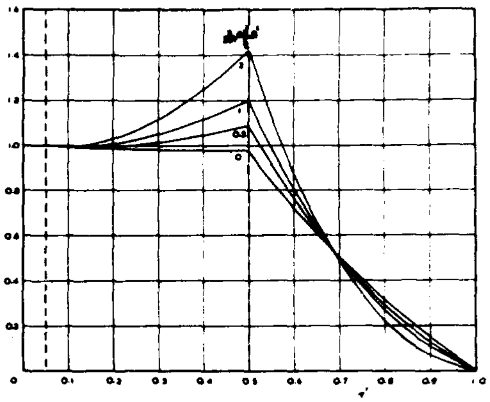

Fig. 3(a) Pressure distribution in a hydrostatic step bearing. $\alpha = 5$. $r_0' = 0.05$. $r_1' = 0.5$

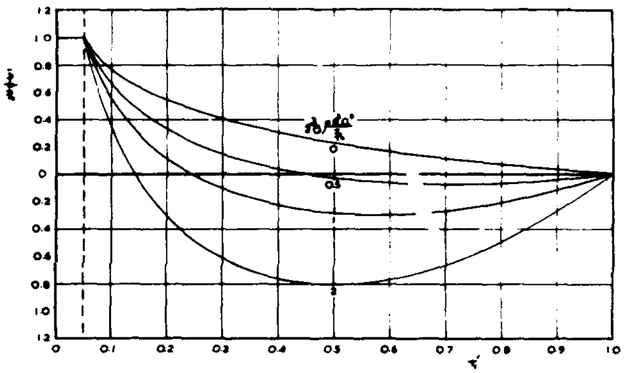

Fig. 3(b) Pressure distribution in a constant film thickness hydrostatic bearing. $\alpha = 1$. $r_0' = 0.05$

Let

$$\bar p = \frac{3}{20}\rho r^2\Omega^2 + C_1 \log_e r + D_1 \qquad r_1 \geq r \geq r_0$$

and

$$\bar p = \frac{3}{20}\rho r^2\Omega^2 + C_2 \log_e r + D_2 \qquad R \geq r \geq r_1$$

For the boundary values let $\bar p = \bar p_0$ when $r = r_0$ and $\bar p = 0$ when $r = R$. For the first condition $\bar p_0$ will be the supply pressure if r_0 is small compared with R.

The additional conditions needed to determine C_1, C_2, D and D_2 are obtained from the requirements of flow and pressure continuity at the step. The expressions obtained are,

$$C_1 = \frac{\bar p_0\left[1 + S(1 - r_0'^2)\right]}{(\alpha^3 - 1)\log_e r_1' + \log_e r_0'}$$

$$C_2 = \frac{\alpha^3 \bar p_0\left[1 + S(1 - r_0'^2)\right]}{(\alpha^3 - 1)\log_e r_1' + \log_e r_0'}$$

$$D_1 = \frac{\bar p_0\left[(\alpha^3-1)\log_e r_1' - \log_e R\right] - S\bar p_0\left[r_0'^2(\alpha^3-1)\log_e r_1' + \log_e r_0' + (1 - r_0'^2)\log_e R\right]}{(\alpha^3-1)\log_e r_1' + \log_e r_0'}$$

$$D_2 = \frac{-\alpha^3 \bar p_0 \log_e R - S\bar p_0\left[(\alpha^3-1)\log_e r_1' + \log_e r_0' + \alpha^3(1 - r_0'^2)\log_e R\right]}{(\alpha^3-1)\log_e r_1' + \log_e r_0'}$$

where

$$S = \frac{3}{20}\frac{\rho R^2\Omega^2}{\bar p_0}$$

The pressure distributions take the form,

$$\frac{\bar p}{\bar p_0} = \frac{\log_e r' + (\alpha^3-1)\log_e r_1'}{(\alpha^3-1)\log_e r_1' + \log_e r_0'} + \frac{S\left[(\alpha^3-1)(r'^2 - r_0'^2)\log_e r_1' + (1 - r_0'^2)\log_e r' - (1 - r'^2)\log_e r_0'\right]}{(\alpha^3-1)\log_e r_1' + \log_e r_0'}$$

$$r_1 \geq r \geq r_0$$

$$\frac{\bar p}{\bar p_0} = \frac{\alpha^3 \log_e r'}{(\alpha^3-1)\log_e r_1' + \log_e r_0'} + \frac{S\left[\alpha^3(1 - r_0'^2)\log_e r' - (1 - r'^2)\left\{(\alpha^3-1)\log_e r_1' + \log_e r_0'\right\}\right]}{(\alpha^3-1)\log_e r_1' + \log_e r_0'} \qquad (12)$$

$$R \geq r \geq r_1$$

Load Capacity

The load carrying capacity of the bearing P can be evaluated from equation (12) and the relationship,

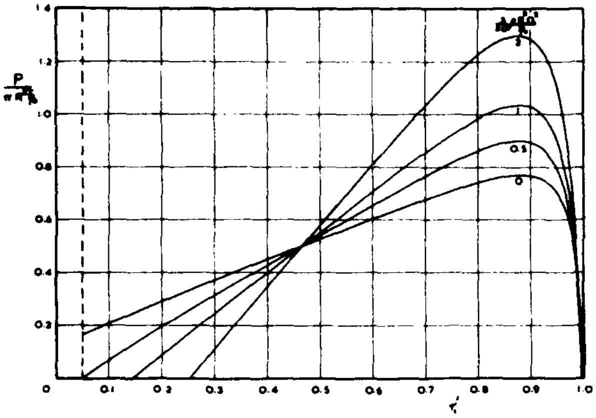

Fig. 4 Variation of load parameter with step position. α = 5. $r_0' = 0.05$

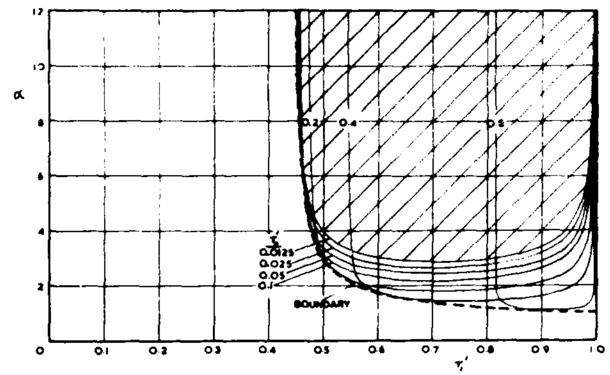

Fig. 5 Contours of r_0' for constant load capacity at all speeds

$$P = \pi r_0^2 \bar{p}_0 + 2\pi \int_{r_0}^{r_1} r \bar{p}\, dr + 2\pi \int_{r_1}^{R} r \bar{p}\, dr$$

The result is,

$$\frac{P}{\pi R^2 \bar{p}_0} = \frac{(\alpha^3-1)(r_1'^2 - r_0'^2) - \alpha^3(1 - r_0'^2)}{2\left[(\alpha^3-1)\log_e r_1' + \log_e r_0'\right]}$$

$$+ S\frac{\left[\left\{(\alpha^3-1)\log_e r_1' + \log_e r_0'\right\}(1+r_0'^2) + (r_1'^2 - r_0'^2) + \alpha^3(1 - r_1'^4)\right](r_0'^2 - 1)}{2\left[(\alpha^3-1)\log_e r_1' + \log_e r_0'\right]} \quad (13)$$

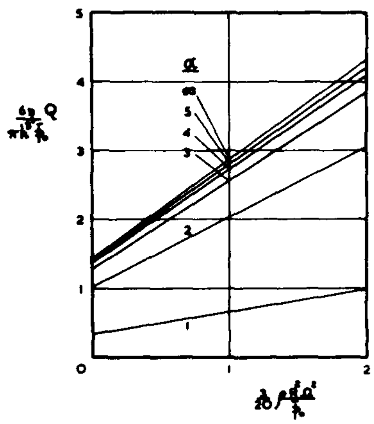

Fig. 6 Variation of flow rate parameter with inertia parameter. $r_0' = 0.05$. $r_1' = 0.5$

Lubricant Flow Rate

The volume rate of the flow of lubricant through the bearing can be found by substituting the appropriate value of C in equation (11).

$$\frac{6\eta Q}{\pi h^3 \bar{p}_0} = -\alpha^3\left[\frac{1 + S(1 - r_0'^2)}{(\alpha^3-1)\log_e r_1' + \log_e r_0'}\right] \quad (14)$$

CALCULATIONS

The pressure distributions represented by equation (12) clearly contain two components. The first group of terms in each equation gives the pressure distribution which is obtained when the bearing surfaces are stationary, whilst the second group represents the modification to the pressure caused by rotation.

In Fig.2 the pressure distributions for a stationary bearing are shown for various film thickness ratios α. The bearing considered has a lubricant supply hole of radius 1/20 of the bearing radius. The solution for parallel surfaces with a constant film thickness (α = 1) reduces to

$$\frac{\bar{p}}{\bar{p}_0} = \frac{\log_e r'}{\log_e r_0'}$$

The pressure distributions as modified by the rotation of one of the bearing components have been calculated for a step bearing (α = 5) and a constant film thickness bearing (α = 1). These results are shown in Fig.3(a) and Fig.3(b), respectively.

The way in which the load carrying capacity varies with the speed of rotation is dependent upon the radius of the step for a bearing of given lubricant supply hole radius. Values of $P/\pi R^2 \bar{p}_0$ have been computed from equation (13) for a film thickness ratio of 5. The values obtained are plotted against the step location parameter r_1' for various values of the inertia parameter S in Fig.4. The relationship between $P/\pi R^2 \bar{p}_0$ and S is linear for a bearing of given geometry. Both bearings have been assumed to have a lubricant supply hole of radius 1/20 of the bearing radius. For the stationary constant film thickness bearing equation (13) becomes

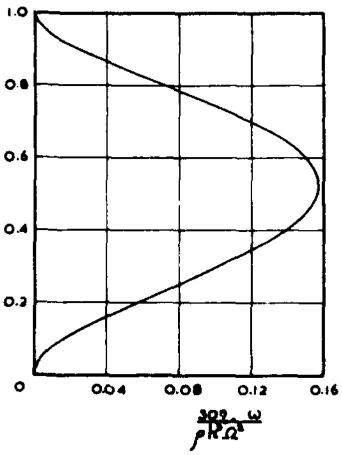

Fig. 7 Radial velocity profiles in a constant film thickness hydrostatic bearing. $r_o' = 0.05$. - - - - no rotation

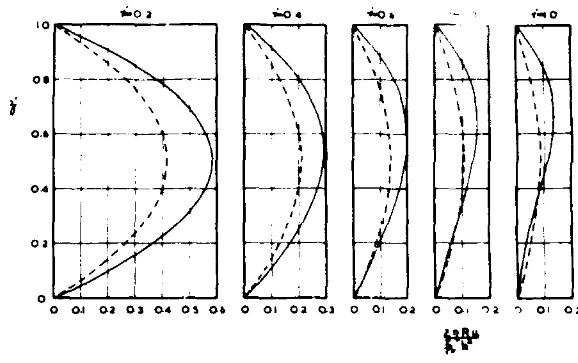

Fig. 8 Axial velocity distribution in a hydrostatic thrust bearing

$$\frac{P}{\pi R^2 \bar{p}_o} = -\frac{(1-r_o'^2)}{2 \log_e r_o'}$$

This result has been presented by Fuller (5).

It can be seen from Fig.4 that if the step is located such that $r_1' = 0.465$ the load capacity of the bearing is independent of speed. If $r_1' < 0.465$ the load capacity will decrease as the speed increases, but if $r_1' > 0.465$ the reverse effect occurs. The value of r_1' required to maintain a constant load capacity at all speeds is dependent upon the film thickness ratio α and r_o'. These null values of r_1' are shown for various values of α and r_o' in Fig.5.

The flow rate parameter $(6\eta Q/\pi h^3 \bar{p}_o)$ has been computed from equation (14). The results are plotted against the inertia parameter S for various values of α in Fig.6. The particular values of r_o' and r_1' selected for this calculation were again 0.05 and 0.5 respectively.

DISCUSSION OF RESULTS

Most hydrostatic thrust bearings are of the step kind shown in Fig.1. The radius of the recess is often appreciable to enable the static supply pressure to separate the bearing elements. For such a bearing it is customary to assume that the pressure at the lubricant supply hole extends to the step. The validity of this assumption is confirmed by the results presented in Fig.2. Even from a modest film-thickness ratio of 5 the pressure at the step only falls below the supply pressure by about 2.6 per cent for the geometry considered.

Perhaps the outstanding feature of the pressure curves shown in Figs.3(a) and 3(b) is the considerable modification which is introduced by inertia effects. For example, when $r_o' = 0.05$, $r_1' = 0.5$ and $\alpha = 5$ a value of unity for S causes the pressure at the step to rise by 22.8 per cent above the pressure obtained with stationary surfaces. A value of unity for the inertia parameter can be obtained in a 6 in. radius bearing supplied with oil at a pressure of 100 psi at a rotational speed of about 4500 rpm.

Although the pressure rise in the region of the step is considerable for the bearing geometry on which Fig.3(a) is based, the load carrying capacity is not greatly affected. The reason for this is that the pressure falls slightly below the values obtained in the absence of rotation at a radius ratio greater than about 0.68. These reduced pressures do of course act over a large proportion of the bearing area.

For a constant film thickness bearing ($\alpha = 1$) rotation always lowers the pressures. An interesting feature of Fig.3(b) is that the pressure in the fluid film falls below the ambient pressure at comparatively small values of the inertia parameter. If these sub-ambient pressures exist in the fluid then the load carrying capacity of such a bearing can rapidly be reduced to zero. The pressure may fall below the saturation pressure to a small extent without the fluid rupturing due to the emergence of air from solution. However due to cavitation the sub-ambient pressures which would be needed to reduce the load capacity to zero are unlikely to exist. If the fluid film is ruptured by cavitation and the pressure maintained above ambient the load will be reduced but it will never become zero. An analysis including this effect would have to employ an appropriate cavitation boundary condition. It should also be recalled that isothermal conditions have been assumed in the analysis. The pressure distribution and load capacity may be affected if thermal effects influence the fluid properties.

Even is sub-ambient pressures are presumed to exist to such an extent that the load carrying capacity of the bearing considered in Fig.3(b) is reduced to zero, the radial velocity of the fluid is not reversed. It is instructive to see the velocity profiles at various radial positions with and without rotation in the constant film thickness bearing.

The velocity distribution can be obtained by substituting $d\bar{p}/dr$ from equation (12) into equation (8). The resulting expression can be written as,

$$\frac{2\eta R}{\bar{p}_0 h^2} u = \frac{z'(z'-1)}{r'\log_e r_0'} + S\left[z'(z'-1)\left\{2r' + \frac{(1-r_0'^2)}{r'\log_e r_0'}\right\} - \frac{10}{9} z'(z'^3-1)r'\right] \quad (15)$$

Now with $r_0' = 0.05$ zero load capacity would be achieved with $S = 0.4986$. The velocity profiles under these conditions are shown in Fig.7. As expected the greatest distortion of the parabolic profile associated with stationary surfaces occurs at large values of the radius. In this region the radial velocity of the fluid is decreased near the stationary surface. However even at $r' = 1$ the velocity is never directed inwards.

Apart from the radial and circumferential velocity components there is a velocity in the axial z-direction when one of the bearing elements is rotating. In lubrication theory the existence of fluid velocities across the oil film is normally associated with problems in which at least one of the bounding solids has a velocity in the z-direction. However in this case transverse velocities exist even though the surfaces are parallel and the film thickness constant.

The continuity equation can be written as,

$$\frac{\partial w}{\partial z} = -\frac{1}{r}\cdot\frac{\partial . ur}{\partial r}$$

When equation (15) is multiplied by r and differentiated with respect to r the first term on the right-hand side vanishes. Thus if the second term is also reduced to zero by considering stationary surfaces we see that $\partial w/\partial z = 0$. Since w is zero at the boundaries it is also zero everywhere. Hence for a static parallel surface thrust bearing the flow is entirely radial.

With rotation the second term does not vanish and hence we find

$$\frac{\partial w}{\partial z'} = \frac{\rho h^3 \Omega^2}{30\eta}\left[5z'^4 - 9z'^2 + 4z'\right]$$

Thus

$$\frac{30\eta}{\rho h^3 \Omega^2} w = \left[z'^5 - 3z'^3 + 2z'^2\right] \quad (16)$$

It will be noted that this expression is independent of the radius. The form of w is shown in Fig.8. Owing to the axial velocity components a fluid element will move towards the moving surface as it spirals through the bearing.

Returning now to the stepped bearing we see from Fig.4 that the nature of the change in load carrying capacity is dependent upon the location of the step. Since it is desirable to ensure that the load capacity of any particular bearing does not decrease as the speed rises a more comprehensive diagram has been presented in Fig.5. The notable result which is obtained from the analysis on which Fig.5 is based is that inertia effects cannot increase the load carrying capacity of any stepped bearing of the form considered if the step is located at a radius less than 0.4508 of the bearing radius.

For a bearing having a value of r_0' of 0.0125 an increase in load capacity will occur as the speed rises if the step location r_1' and film thickness ratio α are selected such that the point lies in the shaded part of Fig.5. Outside this region the load capacity will be reduced. For different values of r_0' the shaded portion of Fig.5 should be extended to the appropriate contour. Although the contours shown in Fig.5 extend to large values of r_0' it should be remembered that the analysis assumes the existence of a constant pressure out to $r' = r_0'$. This condition is unlikely to hold at large values of r_0'. Along each contour the load capacity is constant at all speeds and it is given by the relationship.

$$\frac{P}{\pi R^2 \bar{p}_0} = \frac{1 + r_0'^2}{2}$$

In the region to the left of the boundary shown in Fig.5 the load capacity is always reduced as the speed increases. This result does not depend on the radius of the lubricant supply hole.

Most step bearings operate with a film-thickness ratio greater than 5 and it is clear from Figs.4 and 5 that the greatest benefit can be obtained from inertia effects only if r_1' is made considerably greater than 0.5. The practical limitation on this requirement is of course that the static load of the rotating machinery has to be carried on the annular bearing surface. The theoretical limitation can be seen in Fig.4. As r_1' approaches unity the load capacity starts to fall back towards the constant film thickness value. The theoretical optimum for r_1' will have to be determined with regard to r_0'.

All the results have so far been discussed with a constant supply pressure in mind. The bearing performance cannot however be divorced from the lubricant supply arrangement. Without detailed consideration of any particular lubricant supply system it is possible to see how inertia effects may be responsible for a reduction in oil film thickness as the speed of a hydrostatic bearing is increased.

Consider a step bearing with the step located in a position which, at high speed, results in a decreased load capacity based on a constant supply pressure. As the speed increases equilibrium of the loaded rotating member will be lost if the supply pressure remains constant. The rotating member will therefore move towards the stationary bearing surface. The decrease in film thickness caused by this movement will reduce the flow of lubricant through the bearing and if the oil pump delivery pressure remains constant this will enable the pressure at entry to the bearing to rise. This process will continue until the increased supply pressure can restore the initial load carrying capacity, but the bearing surfaces will now be separated by a smaller gap. This argument can also be applied to show that correct positioning of the step will produce an increase in film thickness as the speed increases.

The analysis has shown that inertia considerations can considerably influence the prediction of hydrostatic thrust bearing performance. The inertia effects are detrimental in the case of constant thickness bearings but they may be advantageous in the case of a stepped bearing if the step is correctly located.

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